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Symmetry content of a generalized p-form model of Schwarz-type in d dimensions

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Abstract. We derive the vector supersymmetry and the Ł-symmetry transformations for the fields of a generalized topological p-form model of Schwarz-type in d space-time dimensions.

1 General setup

BF models and their properties are a widely investigated area in the literature [1]. In particular in ref. [2, 3] BF models in arbitrary dimensions are considered in great detail. In ref. [4] a conclusive formalism in order to derive the minimal action in the framework of Batalin and Vilkovisky (BV) is presented. Based on these concepts ref. [5] introduces a possible generalization to more generic models of Schwarz-type. In this paper we present the BRST-variations, the vector supersymmetry (VSUSY) as well as a scalar supersymmetry, denoted as Ł-symmetry, of the proposed model.
The classical, invariant action of such a generalized p-form model in d space-time dimensions is give by[1]

$$S_{inv} = \int_{\mathcal{M}_d} \left\{ B_p F_2 + \sum_i X_{p_i} D Y_{r_i} \right\}, \tag{1}$$

where $p = d-2$, $r_i = d - p_i - 1$ and $\sum_i$ denotes the summation over an arbitrary set of pairs of p_i- and r_i-form fields. $F_2 = dA + AA$ is the usual curvature of the connection one-form A and the covariant derivative is $D = d + [A, .]$. All commutators $[., .]$ are understood in a graded sense. The fields B_p, X_{p_i} and Y_{r_i} exhibit more or less reducible gauge symmetries, *e. g.* $\delta_{\lambda_{p-1}} B_p = D\lambda_{p-1}$. The algebra of gauge-transformations closes on-shell. The gauge-invariant equations of motion imply the zero-curvature conditions.
Following the guideline of [5] we define pairs of generalized forms which are called dual to each other[2]

$$\begin{array}{rclcrcl} \tilde{B}_p &=& B_d^{-2} + B_{d-1}^{-1} + B_p + \ldots + B^p, & \quad & \tilde{A} &=& A_d^{1-d} + \ldots + A_2^{-1} + A + c, \\ \tilde{X}_{p_i} &=& X_d^{p_i-d} + \ldots + X_{p_i} + \ldots + X^{p_i}, & & \tilde{Y}_{r_i} &=& Y_d^{r_i-d} + \ldots + Y_{r_i} + \ldots + Y^{r_i}. \end{array} \tag{2}$$

For later convenience we cast all fields of *e. g.* $\tilde{X}_{p_i}$ with negative Faddeev-Popov ($\Phi\Pi$) charge into $\check{X}_{p_i}$, whereas the contributions with positive or zero $\Phi\Pi$-charge are collected in $\hat{X}_{p_i}$. Hence, a total expansion is $\tilde{X}_{p_i} = \check{X}_{p_i} + \hat{X}_{p_i}$. The denotation of dual fields becomes more evident if one observes that the fields with negative

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[1]In the following we omit the wedge product sign $\wedge$.
[2]Upper indices label the ghost-number and lower ones denote the form-degree.

$\Phi\Pi$-charge, serve as antifields in the sense of Batalin and Vilkovisky for the elements of the dual partner with positive ghost-degree, *i. e.* $\check{X}_{p_i} = \pm(\hat{Y}_{r_i})^*$, and vice versa[3]. The classical gauge and ghost fields can be addressed by Φ^a, whereas the corresponding antifields are collected in Φ^*_a.

2 BRST-symmetry and BV-action

With the definition of a generalized exterior derivative $\tilde{d} = d + s$, with s denoting the BRST-differential, we can define a generalized covariant derivative $\tilde{D} = \tilde{d} + [\tilde{A}, .]$ and construct the curvatures

$$\tilde{F}_2 = \tilde{d}\tilde{A} + \tilde{A}\tilde{A}, \quad \tilde{G}_{p+1} = \tilde{D}\tilde{B}_p, \quad \tilde{H}_{p_i+1} = \tilde{D}\tilde{X}_{p_i}, \quad \tilde{I}_{r_i+1} = \tilde{D}\tilde{Y}_{r_i}. \tag{3}$$

The BRST-transformations of the fields are determined through so-called horizontality conditions [6] which read

$$\tilde{F}_2 = 0, \quad \tilde{G}_{p+1} = \textstyle\sum_i (-1)^{p_i}[\tilde{X}_{p_i}, \tilde{Y}_{r_i}], \quad \tilde{H}_{p_i+1} = 0, \quad \tilde{I}_{r_i+1} = 0. \tag{4}$$

With the definitions $F_2^{\tilde{A}} = d\tilde{A} + \tilde{A}\tilde{A}$ and $D^{\tilde{A}} = d + [\tilde{A}, .]$ this yields

$$\begin{array}{rclrcl} s\tilde{A} &=& -F_2^{\tilde{A}}, & s\tilde{B}_p &=& -D^{\tilde{A}}\tilde{B}_p + \sum_i (-1)^{p_i}[\tilde{X}_{p_i}, \tilde{Y}_{r_i}], \\ s\tilde{X}_{p_i} &=& -D^{\tilde{A}}\tilde{X}_{p_i}, & s\tilde{Y}_{r_i} &=& -D^{\tilde{A}}\tilde{Y}_{r_i}, \end{array} \tag{5}$$

The above BRST-transformations admit the cocycle equation

$$\tilde{d}\,\mathrm{tr}\left(\tilde{B}_p F_2^{\tilde{A}} + \sum_i \tilde{X}_{p_i} D^{\tilde{A}} \tilde{Y}_{r_i}\right) = 0. \tag{6}$$

This leads to the BRST-invariant, minimal BV-action

$$S_{min} = \int_{\mathcal{M}_d} \left\{ \tilde{B}_p F_2^{\tilde{A}} + \sum_i \tilde{X}_{p_i} D^{\tilde{A}} \tilde{Y}_{r_i} \right\}\Bigg|_d^0 . \tag{7}$$

An expansion in the generic fields yields

$$\begin{aligned} S_{min} \;&=\; S_{inv} + \int_{\mathcal{M}_d} \Bigg\{ \check{B}_p(-s\hat{A}) + \check{A}(-s\hat{B}_p) + \sum_i \Big(\check{X}_{p_i}(-s\hat{Y}_{r_i}) + (-1)^{d(p_i+1)}\check{Y}_{r_i}(-s\hat{X}_{p_i}) \Big) \\ &+\; \sum_i (-1)^{p_i} \check{A} \Big([\hat{X}_{p_i}, \check{Y}_{r_i}] + [\check{X}_{p_i}, \hat{Y}_{r_i}] \Big) - \frac{1}{2}\hat{B}_p[\check{A}, \check{A}] \Bigg\}\Bigg|_d^0 . \end{aligned} \tag{8}$$

The latter action induces the antifield identification

$$\check{B}_p = (\hat{A})^*, \quad \check{A} = (\hat{B}_p)^*, \quad \check{X}_{p_i} = (\hat{Y}_{r_i})^*, \quad \check{Y}_{r_i} = (-1)^{d(p_i+1)}(\hat{X}_{p_i})^*, \tag{9}$$

in order to ensure coincidence with the BRST-transformations obtained from

$$s\Phi^*_a = -\frac{\delta S_{min}}{\delta \Phi^a}, \qquad s\Phi^a = -\frac{\delta S_{min}}{\delta \Phi^*_a}. \tag{10}$$

3 Gauge-fixing

Similarly to the BF model we need to introduce for each classical gauge field a BV-pyramid (*c. f.* Table 1 and 2). From the gauge-fixing point of view the dual fields B_p and A can be considered as an ordinary p- and one-form field. Hence, we define $\tilde{X}_{d-2} \equiv \tilde{B}_p$ and $\tilde{Y}_1 \equiv \tilde{A}$ and according definitions for the antighost and multiplier fields. Moreover, for the sake of a compact notation when giving the gauge-fermion, we let the antighost fields to the lowest order n be the classical gauge and ghost fields, *i. e.* $\bar{v}^q_{p_i-q} \equiv X^q_{p_i-q}$ and $\bar{w}^q_{r_i-q} \equiv Y^q_{r_i-q}$. The gauge-fixing fermion Ψ_{gf} then looks like

[3]The superscript * denotes antifields.

$$\begin{array}{ccccccc} & & & X_{p_i} & & & \\ & & \bar{v}^{-1}_{p_i-1} & & X^{1}_{p_i-1} & & \\ & \bar{v}^{-2}_{p_i-2} & & \bar{v}_{p_i-2} & & X^{2}_{p_i-2} & \\ \cdots & & & & & & \cdots \end{array}$$

Table 1: X_{p_i} *pyramid*

$$\begin{array}{ccccccc} & & & Y_{r_i} & & & \\ & & \bar{w}^{-1}_{r_i-1} & & Y^{1}_{r_i-1} & & \\ & \bar{w}^{-2}_{r_i-2} & & \bar{w}_{r_i-2} & & Y^{2}_{r_i-2} & \\ \cdots & & & & & & \cdots \end{array}$$

Table 2: Y_{r_i} *pyramid*

$$\begin{aligned} \Psi_{gf} &= \int_{\mathcal{M}_d} \Bigg\{ \sum_i \sum_{n=1}^{p_i} \sum_{q=0}^{p_i-n} \bar{v}^{\gamma(n)}_{p_i-q-n} \left({}^n\alpha_{(q)} * \kappa^{\gamma(n+1)}_{p_i-q-n} + d * \bar{v}^{\gamma(n+1)}_{p_i-q-n+1} \right) \\ &+ \sum_i \sum_{n=1}^{r_i} \sum_{q=0}^{r_i-n} \bar{w}^{\gamma(n)}_{r_i-q-n} \left({}^n\beta_{(q)} * \lambda^{\gamma(n+1)}_{r_i-q-n} + d * \bar{w}^{\gamma(n+1)}_{r_i-q-n+1} \right) \Bigg\}, \end{aligned} \tag{11}$$

where $*$ is the Hodge-star operator, ${}^n\alpha_{(q)}, {}^n\beta_{(q)} \in \mathbf{R}$ are some arbitrary parameters and $\gamma(n = 2k) = q$ or $\gamma(n = 2k+1) = -q-1$. The implementation of external sources ρ^*_a implies a further contribution to the gauge-fermion

$$\Psi_{ext} = \int_{\mathcal{M}_d} \sum_i (-1)^{(d+1)p_i} \left(\sum_{q=0}^{p_i} (-1)^{d+1} X^q_{p_i-q} \tau^{*-q-1}_{d-p_i+q} + \sum_{q=0}^{r_i} Y^q_{r_i-q} \eta^{*-q-1}_{d-r_i+q} \right).$$

The total gauge-fermion is $\Psi = \Psi_{gf} + \Psi_{ext}$. The corresponding multiplier fields to the antighosts $\bar{v}^q_p$ and $\bar{w}^q_p$ are κ^{q+1}_p and λ^{q+1}_p respectively. The auxiliary contribution is given by

$$S_{aux} = -\int_{\mathcal{M}_d} \sum_i \left(\sum_{n=1}^{p_i} \sum_{q=0}^{p_i-n} \left(\bar{v}^{\gamma(n)}_{p_i-q-n} \right)^* \kappa^{\gamma(n)+1}_{p_i-q-n} + \sum_{n=1}^{r_i} \sum_{q=0}^{r_i-n} \left(\bar{w}^{\gamma(n)}_{r_i-q-n} \right)^* \lambda^{\gamma(n)+1}_{r_i-q-n} \right).$$

The BRST-doublet fields are collected in $(\bar{C}^\alpha, \Pi^\alpha)$ and together with the fields of Φ^a they may by addressed by Φ^A. The non-minimal solution of the BV-masters equation is given by $S_{nm} = S_{min} + S_{aux}$. The antifields can be expressed as functionals of the fields via the equation

$$\Phi^*_A[\Phi^A] = (-1)^{(d+1)|\Phi^A|+d} \frac{\delta \Psi}{\delta \Phi^A}. \tag{12}$$

This admits the elimination of the antifields of the action in order to get

$$\Gamma^{(0)} = S_{nm}|_{\Phi^*_A[\Phi^A]} = S_{inv} + S_{gf} + S_{ext}, \tag{13}$$

where $S_{gf} = s\Psi_{gf} + S^{mod}_{gf}$ and $S_{ext} = s\Psi_{ext} + S^{mod}_{ext}$. The additional contribution $S^{mod} = S^{mod}_{gf} + S^{mod}_{ext}$ appears since the BRST-transformations of the fields (5) exhibit an antifield-dependency. The structure of S^{mod} can be seen from the last line in (8). Although, S^{mod} can not be written as a BRST-exact expression, it however, does not spoil the topological character of the model, due to its metric independence. With equation (12) the antifields can also be eliminated in the generalized field expansions (2).

4 Vector supersymmetry

4.1 Derivation

The above concepts provide a neat formalism to derive the VSUSY [7], $\delta_\tau = \tau^\mu \delta^{-1}_\mu$, where τ is a constant, BRST-invariant, even graded vector-field[4]. The VSUSY-transformations satisfy the algebra

$$[s, \delta_\tau] = \mathcal{L}_\tau = [d, i_\tau], \tag{14}$$

[4]Henceforth $\mathcal{M}_d$ is considered as a flat space-time manifold.

where $\mathcal{L}_\tau$ is the Lie-derivative and i_τ is the interior product along τ. The algebra (14) suggests an equivalence between i_τ and δ_τ, yielding for the δ_τ-transformations

$$\delta_\tau \tilde{X}_{p_i} = i_\tau \tilde{X}_{p_i}, \quad \delta_\tau \tilde{Y}_{r_i} = i_\tau \tilde{Y}_{r_i}. \tag{15}$$

This determines the VSUSY-transformations of the classical gauge and ghost fields but also of the antighost fields at the first reducibility level. However, in order to describe the δ_τ-transformations of the higher reducibility antighost fields we need to collect also the antighosts with positive $\Phi\Pi$-charge together with their corresponding antifields in generalized forms[5]

$$\hat{\bar{v}}_{p_i-n} = \sum_{q=0}^{p_i-n} \bar{v}^q_{p_i-n-q}, \qquad (\hat{\bar{v}}_{p_i-n})^* = \sum_{q=0}^{p_i-n} (\bar{v}^q_{p_i-n-q})^*, \qquad n = 2k. \tag{16}$$

Although only the antighosts with positive $\Phi\Pi$-charge are cast into this scheme, the antighost fields with negative ghost-degree come into play automatically via the elimination of the antifields (12). The VSUSY-transformations now follow from the proposed equivalence of the δ_τ-operation and the interior product i_τ in the space of generalized forms and their duals, thus we get

$$\delta_\tau \hat{\bar{v}}_{p_i-n} = i_\tau \hat{\bar{v}}_{p_i-n}, \qquad \delta_\tau (\hat{\bar{v}}_{p_i-n})^* = i_\tau (\hat{\bar{v}}_{p_i-n})^*. \tag{17}$$

This determines the VSUSY-transformations of the remaining antighost fields, but also the gauge-parameters ${}^n\alpha_{(q)} = {}^n\beta_{(q)} = (-1)^d$ for $n = 2k$.

4.2 Explicit results

The detailed results for the elements of Φ^a yield from (15)

$$\begin{array}{rcll} \delta_\tau X_{p_i} &=& -i_\tau \left(\eta^{*-1}_{p_i+1} - (-1)^{(d+1)p_i+d} * d\bar{w}^{-1}_{r_i-1} \right), & \\ \delta_\tau X^q_{p_i-q} &=& i_\tau X^{q-1}_{p_i-q+1}, & q = 1, \dots, p_i, \\ \delta_\tau Y_{r_i} &=& -i_\tau \left(\tau^{*-1}_{r_i+1} - (-1)^{d+p_i+1} * d\bar{v}^{-1}_{p_i-1} \right), & \\ \delta_\tau Y^q_{r_i-q} &=& i_\tau Y^{q-1}_{r_i-q+1}, & q = 1, \dots, r_i. \end{array} \tag{18}$$

The antighost field transformations are determined through (17)

$$\begin{array}{rcll} \delta_\tau \bar{v}_{p_i-n} &=& 0, & \\ \delta_\tau \bar{v}^q_{p_i-q-n} &=& i_\tau \bar{v}^{q-1}_{p_i-q-n+1}, & q = 1, \dots, p_i - n, \\ \delta_\tau \bar{v}^{-q-1}_{p_i-q-n} &=& (-1)^{d+p_i+q+1} g(\tau) \bar{v}^{-q-2}_{p_i-q-n-1}, & q = 0, \dots, p_i - n - 1, \\ \delta_\tau \bar{v}^{-p_i+n-1}_0 &=& 0, & \end{array} \tag{19}$$

and

$$\begin{array}{rcll} \delta_\tau \bar{w}_{r_i-n} &=& 0, & \\ \delta_\tau \bar{w}^q_{r_i-q-n} &=& i_\tau \bar{w}^{q-1}_{r_i-q-n+1}, & q = 1, \dots, r_i - n, \\ \delta_\tau \bar{w}^{-q-1}_{r_i-q-n} &=& (-1)^{d+r_i+q+1} g(\tau) \bar{w}^{-q-2}_{r_i-q-n-1}, & q = 0, \dots, r_i - n - 1, \\ \delta_\tau \bar{w}^{-r_i+n-1}_0 &=& 0, & \end{array} \tag{20}$$

where the Hodge-star $*$ intertwines between the interior product and the one-form $g(\tau) = \tau_\mu dx^\mu$ in the way $i_\tau * \alpha_p = (-1)^p * g(\tau)\alpha_p$. The corresponding multiplier field transformations can be obtained from the algebra (14) through

$$\delta_\tau \Pi^\alpha = \mathcal{L}_\tau \bar{C}^\alpha - s\delta_\tau \bar{C}^\alpha, \tag{21}$$

for some element of the BRST-doublets $(\bar{C}^\alpha, \Pi^\alpha)$. Furthermore, the δ_τ-variations of the external sources are

$$\begin{array}{rcll} \delta_\tau \tau^{*-q-1}_{d-p_i+q} &=& i_\tau \tau^{*-q-2}_{d-p_i+q+1}, & q = 0, \dots, p_i - 1, \\ \delta_\tau \tau^{*-p_i-1}_d &=& 0, & \\ \delta_\tau \eta^{*-q-1}_{d-r_i+q} &=& i_\tau \eta^{*-q-2}_{d-r_i+q+1}, & q = 0, \dots, r_i - 1, \\ \delta_\tau \eta^{*-r_i-1}_d &=& 0. & \end{array} \tag{22}$$

[5] The reasoning is only given for the X_{p_i}-sector, but the same arguments hold for the antighost fields $\bar{w}^q_{r_i-n-q}$.

The algebra (14) of the VSUSY-transformations of the classical gauge fields closes on-shell

$$[s,\delta_\tau]\,X_{p_i} = \mathcal{L}_\tau X_{p_i} - (-1)^{d(p_i+1)} i_\tau \frac{\delta\Gamma^{(0)}}{\delta Y_{r_i}}, \qquad [s,\delta_\tau]\,Y_{r_i} = \mathcal{L}_\tau Y_{r_i} - i_\tau \frac{\delta\Gamma^{(0)}}{\delta X_{p_i}}. \tag{23}$$

In general we describe the symmetry content of a model with a Ward-operator

$$\mathcal{W}^I = \int_{\mathcal{M}_d} \sum_\varphi \delta^I \varphi \frac{\delta}{\delta\varphi}, \tag{24}$$

where $\delta^I\varphi$ denotes the field-transformations under the symmetry I and φ stands for all fields characterizing the model in question. In this sense we define a Ward-operator $\mathcal{W}_\tau$ according to the above δ_τ-transformations (18)–(22). By choosing the remaining gauge-parameters ${}^{2k+1}\alpha_{(q)} = {}^{2k+1}\beta_{(q)} = 0$, the application of $\mathcal{W}_\tau$ onto $\Gamma^{(0)}$ leads to a linear breaking term in the quantum fields

$$\mathcal{W}_\tau \Gamma^{(0)} = \Delta_\tau, \tag{25}$$

where

$$\begin{aligned} \Delta_\tau \;=\; & \int_{\mathcal{M}_d} \sum_i (-1)^{p_i} \Bigg\{ (-1)^{d+1} \left(\sum_{q=0}^{p_i} \tau^{*-q-1}_{d-p_i+q} \mathcal{L}_\tau X^q_{p_i-q} + \kappa_{p_i-1} d * i_\tau \eta^{*-1}_{p_i+1} \right) \\ & + \left(\sum_{q=0}^{r_i} \eta^{*-q-1}_{d-r_i+q} \mathcal{L}_\tau Y^q_{r_i-q} + \lambda_{r_i-1} d * i_\tau \tau^{*-1}_{r_i+1} \right) \Bigg\}. \end{aligned} \tag{26}$$

5 Ł-symmetry

5.1 General setup

For the following section we assume that the action is complete in the sense, that it contains all possible types of p-forms that are allowed, but where no particular p-form shall occur more than once. The definitions $I_1^{\tilde{Y}_0} = D^{\tilde{A}}\tilde{Y}_0$, $I_2^{\tilde{Y}_1} \equiv F_2^{\tilde{A}}$, $I_3^{\tilde{Y}_2} = D^{\tilde{A}}\tilde{Y}_2, \ldots, I_r^{\tilde{Y}_{r-1}} = D^{\tilde{A}}\tilde{Y}_{r-1}$ admit to write (7) as

$$S_{min} = \int_{\mathcal{M}_d} \left\{ \sum_{i=1}^r \tilde{X}_{d-i} I_i^{\tilde{Y}_{i-1}} \right\} \Bigg|_d^0 . \tag{27}$$

The upper limit is given by $r = d/2$ for even or $r = (d+1)/2$ for odd dimensions. We define a scalar transformation Ł with ΦΠ-charge -1 with the algebra $[\text{Ł}, s] = 0$. The Ł-transformations act on the generalized fields as follows

$$\begin{array}{rcll} \text{Ł}\tilde{X}_{d-i} &=& \tilde{X}_{d-i-1}, & i = 1,\ldots,r-1, \\ \text{Ł}\tilde{X}_{d-r} &=& 0, & \\ \text{Ł}\tilde{Y}_0 &=& 0, & \\ \text{Ł}\tilde{Y}_{i-1} &=& (-1)^{d+i}\tilde{Y}_{i-2}, & i = 2,\ldots,r. \end{array} \tag{28}$$

5.2 Explicit results

The elimination of the antifields via (12) yields the explicit transformation properties. The classical fields transform as

$$\begin{array}{rcll} \text{Ł}X_{d-i} &=& -\eta^{*-1}_{d-i} + (-1)^{(d+1)i+1} * d\bar{w}^{-1}_{i-1}, & i = 1,\ldots,r-1, \\ \text{Ł}X_{d-r} &=& 0, & \\ \text{Ł}Y_0 &=& 0, & \\ \text{Ł}Y_{i-1} &=& (-1)^{(d+1)i} \left(-\tau^{*-1}_{i-1} + (-1)^{(d+1)i+d} * d\bar{v}^{-1}_{d-i} \right), & i = 2,\ldots,r. \end{array} \tag{29}$$

The ghosts vary under the Ł-symmetry like

$$
\begin{array}{rcllr}
ŁX^q_{d-i-q} & = & X^{q-1}_{d-i-q}, & i=1,\ldots,r-1, & q=1,\ldots,d-i, \\
ŁX^q_{d-r-q} & = & 0, & & q=1,\ldots,d-r, \\
ŁY^q_{i-1-q} & = & (-1)^{d+i}Y^{q-1}_{i-1-q}, & i=2,\ldots,r, & q=1,\ldots,i-1.
\end{array}
\tag{30}
$$

The antighost fields transform as

$$
\begin{array}{rcllr}
Ł\bar{v}_{d-i-n} & = & 0, & i=1,\ldots,r, & \\
Ł\bar{v}^q_{d-i-n-q} & = & \bar{v}^{q-1}_{d-i-n-q}, & i=1,\ldots,r-1, & q=1,\ldots,d-i-n, \\
Ł\bar{v}^q_{d-r-n-q} & = & 0, & & q=1,\ldots,d-r-n, \\
Ł\bar{v}^{-q-1}_{d-1-n-q} & = & 0, & & q=0,\ldots,d-1-n \\
Ł\bar{v}^{-q-1}_{d-i-n-q} & = & (-1)^{i+1}\bar{v}^{-q-2}_{d-i-n-q}, & i=2,\ldots,r, & q=0,\ldots d-i-n,
\end{array}
\tag{31}
$$

and

$$
\begin{array}{rcllr}
Ł\bar{w}_{i-1-n} & = & 0, & i=1,\ldots,r, & \\
Ł\bar{w}^q_{i-1-n-q} & = & (-1)^{d+i}\bar{w}^{q-1}_{i-1-n-q}, & i=2,\ldots,r, & q=1,\ldots i-1-n, \\
Ł\bar{w}^{-q-1}_{i-1-n-q} & = & -\bar{w}^{-q-2}_{i-1-n-q}, & i=1,\ldots,r-1, & q=0,\ldots .i-1-n, \\
Ł\bar{w}^{-q-1}_{r-1-n-q} & = & 0, & & q=0,\ldots,r-1-n.
\end{array}
\tag{32}
$$

Due to $[Ł, s]=0$ the corresponding multiplier fields transform as

$$
Ł\Pi^\alpha = -sŁ\bar{C}^\alpha, \tag{33}
$$

for some element of the BRST-doublets $(\bar{C}^\alpha, \Pi^\alpha)$. Finally, the Ł-variations of the external sources ρ^*_a are

$$
\begin{array}{rcllr}
Ł\tau^{*-q-1}_{1+q} & = & 0, & & q=0,\ldots,d-2, \\
Ł\tau^{*-q-1}_{i+q} & = & (-1)^i\tau^{*-q-2}_{i+q}, & i=2,\ldots,r, & q=0,\ldots,d-i-1, \\
Ł\eta^{*-q-1}_{d-i+1+q} & = & \eta^{*-q-2}_{d-i+1+q}, & i=1,\ldots,r-1, & q=0,\ldots,i-2, \\
Ł\eta^{*-q-1}_{d-r+1+q} & = & 0, & & q=0,\ldots,r-2.
\end{array}
\tag{34}
$$

The algebra $[Ł, s]=0$ on the classical fields is valid on-shell

$$
[Ł, s]\, X_{d-i} = (-1)^{d(i+1)}\frac{\delta\Gamma^{(0)}}{\delta Y_i}, \qquad [Ł, s]\, Y_{i-1} = (-1)^{d+i}\frac{\delta\Gamma^{(0)}}{\delta X_{d-i+1}}. \tag{35}
$$

The inclusion of the external sources also yields a linear breaking of the Ward-identity (*c.f.* (24)) associated to the latter presented Ł-transformations (29)–(34)

$$
\mathcal{W}^Ł\Gamma^{(0)} = \Delta^Ł, \tag{36}
$$

where

$$
\Delta^Ł \;=\; \int_{\mathcal{M}_d}\left\{\sum_{i=1}^{r-1}(-1)^{i+1}\kappa_{d-i-1}d*\eta^{*-1}_{d-i} + \sum_{i=2}^{r}(-1)^{d(i+1)}\lambda_{i-2}d*\tau^{*-1}_{i-1}\right\}. \tag{37}
$$

The symmetry property of the above transformations can also be understood in another context. The action (27) can be seen as the dimensional reduction of a similar model in $d+1$ space-time dimensions. In this sense the Ł-symmetry is nothing else but the leftover of the surplus VSUSY-transformation in the reduced direction. Hence, it is obvious that Ł is indeed a symmetry transformation.

6 Conclusion

In this short note we applied the procedure of [4] to the case of an arbitrary topological p-form model of Schwarz-type in d space-time dimensions [5]. We presented the BRST-transformations in terms of generic form fields, and we extended the formalism to the derivation of the VSUSY-transformations and a scalar supersymmetry called Ł-symmetry.
Obviously, the model under consideration incorporates an ordinary BF theory in arbitrary dimensions [2, 3] by setting all fields except $B_p \equiv X_{d-2}$ and $A \equiv Y_1$ to zero. As a three-dimensional application of the generic p-form model one can reconstruct the results for the so-called BFK model [8]. With a similar method the authors of [9] analyzed the BFK model enlarged by a Chern-Simons term.

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